Educational Background Oriented Schlieren based on a Matlab App and a smartphone camera

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ABSTRACT

Teaching fluid dynamics could be made easier if students were able to directly observe turbulent flows. Background Oriented Schlieren (BOS) could be a well-suited technique to serve that purpose because it does not require seeding or hazardous light source and it is easy to implement. To fully comply with educational constrains the experimental setup must be affordable. In this contribution we test the performances of a low-cost BOS system based on the combination of a smartphone camera together with comBOS, an open-source Matlab toolbox dedicated to performing BOS flow visualization. The flow on which those tools are tested is the thermal plume generated by natural convection above a heated square cylinder. In order to assess the accuracy of the new toolbox, the displacement maps computed by comBOS from images captured by a scientific camera are compared with those obtained by the commercial software Davis 10. Displacements in the range of ± 0.15 pixel obtained from the images of the thermal plume are found nearly identical with both methods. Results only differ by the noise level that was found to be close to 8×10^{-3} pixel with comBOS, 33% higher than with Davis 10. The scientific camera is then replaced by three OPPO A94 smartphones allowing to visualize the plume from three different view angles. The qualitative analysis of the displacement maps obtained from the smartphone cameras and the scientific camera does not reveal any significant difference. Spectra on the fluctuations in time of the angular deflection measured with the two types of cameras are also agreeing well. The possibility of building a low-cost BOS apparatus is therefore demonstrated with a success that exceeds our expectations. Tests were extended by testing thirteen more smartphones of different types. Nine of them were also well-suited for BOS, whereas four are producing non-usable images.

1. Introduction

The success of *An Album of Fluid Motion* by Van Dyke (1982) is a striking demonstration of the need for graphically illustrating concepts in fluid mechanics, especially for teaching purposes. Although flow simulations can meet this desire, only large eddy and direct numerical simulations would be able to highlight the presence of turbulence in flows, although these simulations are hardly achievable by students because of the computing resources they require. Conventional flow visualisation techniques such as particle image velocimetry (PIV) also have their own drawbacks when considered for lab sessions. In particular, concerns about laser risks or hardware and software costs may refrain the use of these techniques. In that sense Background Oriented Schlieren (BOS) is a candidate as an affordable and safe measurement method for illustrating features in many flow configurations. Contrary to conventional Schlieren techniques that require large optical components for beam colimation and focussing, BOS only rests upon the use of a camera and a background image displaying a random pattern referred to as the speckle. The gradients of refractive index in the medium of interest are a-posteriori computed through image processing. For that reason it was initially refereed to as "syntethic Schlieren" (Dalziel et al., 2000). Soon after this, "Background Oriented" was first mentioned (Raffel et al., 2000) together with three statements that still stand; the technique is mostly qualitative; three-dimensional density measurements can be performed providing multiple cameras are operated synchronously; large fields of view can be observed without increasing practical difficulties. For further reading about the theoretical aspects, one may refer the extensive literature adressing this topic, and for instance Raffel (2015) for classical BOS or Nicolas et al. (2016) for three-dimensional BOS.

The cost of commercial solutions for BOS image acquisition and processing is high, and even though they may guarantee a level of quality, affordability is often crucial for teaching application. The aim of this contribution to address the concern of the cost of the BOS measurement technique by demonstrating its feasibility with low cost components. For that purpose, the Matlab application comBOS has been developed by the authors for the processing of BOS images and the generation of the speckle and is freely available on the Matlab community file exchange. For similar cost-reduction objectives, image acquisition may be carried out with smartphone cameras. Smartphone cameras indeed demonstrated their capabilities in experimental fluid dynamics in numerous studies (Hayasaka et al., 2018; Hayasaka & Tagawa, 2019; Minichiello et al., 2021; Käufer et al., 2021; Cierpka et al., 2016). The camera mainly used in this contribution is that of a OPPO A94 smartphone but perspectives of applicability to a diversity of smartphones are also discussed.

In this contribution we compare this combination of an open-source processing software and smartphone camera with a commercial system from LaVision. The comparison is performed on the visualisation of the flow emerging from natural convection around a heated square cylinder.

The features of the Matlab application comBOS are described first in Sec. 2. The experimental setup and procedures are then described in Sec. 3.

Qualitative and quantitative comparisons between the commercial and the low-cost apparatuses are then proposed in Secs. 4 and 5 before a discussion and a conclusion are proposed.

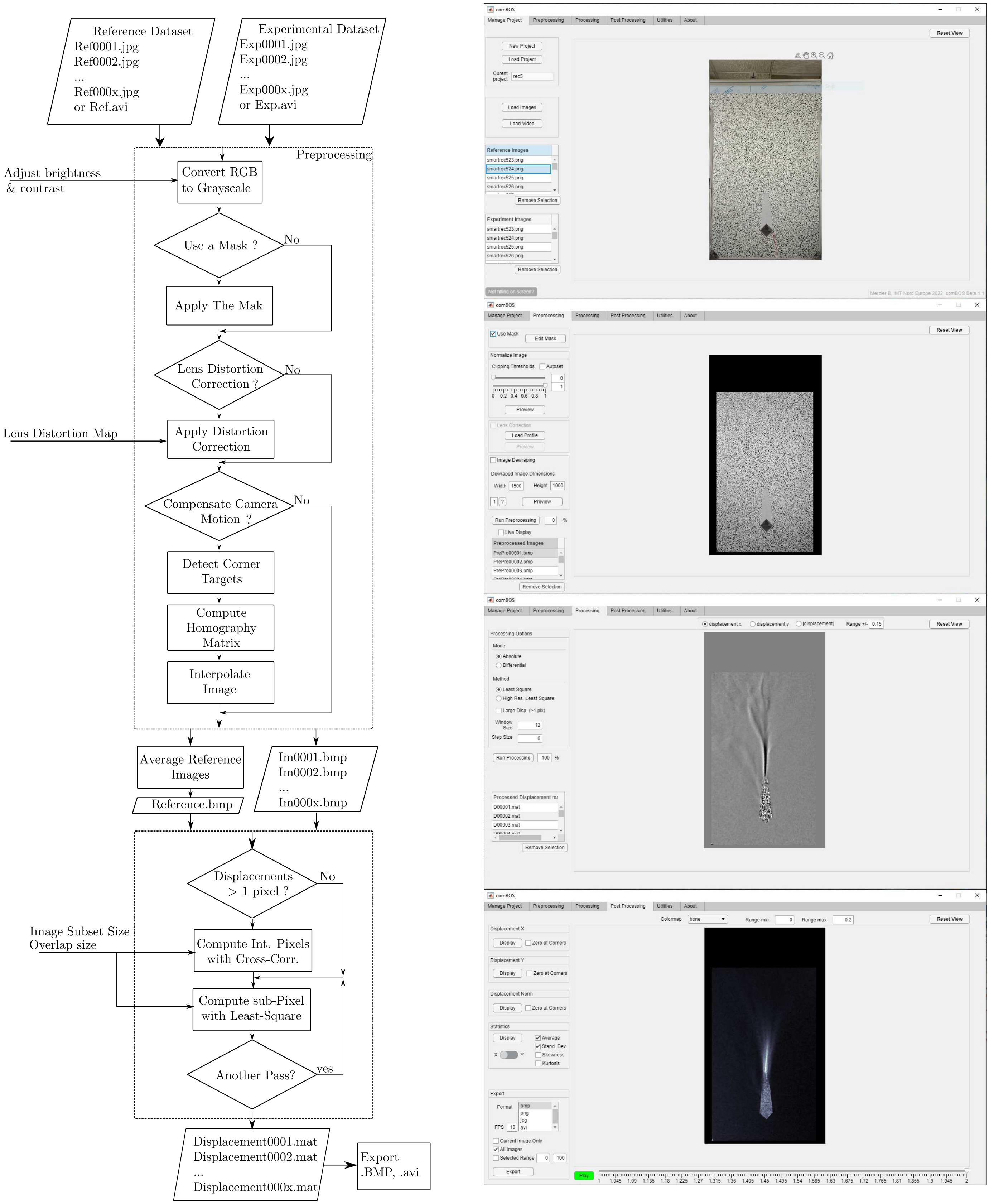

Figure 1. Flow-chart of the application for computing displacements from a set of reference images and experimental images. Screenshots of the application show the four main tabs.

2. Overview of comBOS

The application comBOS was developed with Matlab to provide the community with a user friendly tool with a graphical user interface for BOS experiments. It includes a processing method specifically optimized for BOS images which is significantly less computationally demanding than conventional PIV based algorithm commonly used in BOS processing.

ComBOS is designed with a logic similar to that of typical image processing softwares for PIV, or BOS. This is illustrated by the screenshots of the app together and the flow chart provided in Fig. 1.

• A first tab is for project management and importing images from a folder on the computer or from a video to the project folder.

• A second tab offers preprocessing functionalities. At this step, colour images are converted into gray-level bitmap with an opportunity to optimise the histogram of the gray level. A mask can also be applied to restrict the area of the image considered for computing the displacements. Another function of the preprocessing tab is the possibility of detecting corners of the background speckle to compute the homography matrix and therefore to be able to compensate the effects of the perspective. This functionality was designed to allow performing BOS measurements with phones supported by an unstable stand, or even with hands. It corresponds to the non-ideal situation of a student conducting an experiment at home, for instance during the lab-session of an online course. Implementing this feature raised a need for a correction of the lens distortion, the robustness of which is currently still being tested and thus not included in this contribution.

• The displacement maps are computed in the third tab from the reference image and the experimental images obtained at the preprocessing step. The algorithm for computing displacements constitutes the most important feature of the application. Indeed, to the knowledge of the authors there is no open source code specifically designed for the calculation of BOS displacements. Even though PIV codes based on image cross-correlations can be used, more efficient methods can be implemented for BOS. Indeed, images are only hardly distorted in BOS because pixels displacements are commonly in the order of 10^{-3} to 10^{0} pixel. Direct computation of the displacement can therefore be performed using a linear approximation of the distortion and least square optimisation. The procedure used for this code was adapted from a more general procedure described in Pan et al. (2009), and its following description is supported by figure 2.

First, similarly to PIV, both the reference image and the deformed images are divided into subsets in which displacements are calculated. Let $f(x_i, y_i)$ and $g(x_i, y_i)$ be the gray level maps of the pixels in the same subset at position (x_i, y_i) in the reference image and the deformed image respectively. Assuming a solid body deformation of the image, one wants to determine the displacement Δx, and Δy such that

$$af(x_i, y_i) + b = g(x_i + \Delta x, y_i + \Delta y) \quad (1)$$

with a and b being coefficients for scaling changes in gray level intensity. The objective is therefore to find a, b, Δx and Δy such that

$$g(x_i + \Delta x, y_i + \Delta y) - af(x_i, y_i) - b = 0 \tag{2}$$

The efficiency of the method arises when the Taylor expansion F_i of equation (2) is written for each pixel i of the subset

$$F_i(p) = g_i + \frac{\partial g_i}{\partial x}\Delta x + \frac{\partial g_i}{\partial y}\Delta y - af_i - b \simeq 0 \tag{3}$$

with x and y the horizontal and vertical direction and $p = [\Delta x \; \Delta y \; a \; b]^T$. First order derivatives of the gray levels are computed for each pixel using fourth-order accuracy finite differences.

If there is more than 4 pixels in the subset, p can be determined using a linear least square method with

$$p = (X^T X)^{-1} X^T B \tag{4}$$

where

$$X = \begin{pmatrix} \frac{\partial g_1}{\partial x} & \frac{\partial g_1}{\partial y} & -f_1 & -1 \\ \frac{\partial g_2}{\partial x} & \frac{\partial g_2}{\partial y} & -f_2 & -1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial g_n}{\partial x} & \frac{\partial g_n}{\partial y} & -f_n & -1 \end{pmatrix}, \; B = \begin{pmatrix} -g_1 \\ -g_2 \\ \vdots \\ -g_n \end{pmatrix} \tag{5}$$

The tests performed with synthetic images demonstrated that this simple method is accurate enough to keep any bias within the noise level for displacements up to 0.3 pixel. Larger displacements become underestimated by a few percent. For that reason, an option referred to as "high resolution" can be selected in the application, and turns on a second pass in displacement calculation. This second pass is performed with the same method as the first pass, but the subset of the deformed image is first linearly interpolated on a grid shifted by the formerly calculated Δx and Δy. This second pass is found to decrease the bias to an insignificant level, but it also increases the noise level by approximately 50%.

In addition, displacements larger than one pixel cannot be addressed by this linear method, thus if such large displacements are expected to occur, a first pass based on the classical cross-correlation method can be applied to compute integer pixel displacements. Once this is done, the standard linear method is applied onto subsets shifted in space accordingly to the result of the cross-correlation.

• The last tab is about postprocessing. It contains exporting options for the displacement maps, and vector statistics.

In addition to the main features dedicated to the computation of image displacements described above, comBOS also includes modules for generating the reference background pattern, calibrating the camera distortion, and remotely acquire images from a smartphone providing a third party application is installed on the phone.

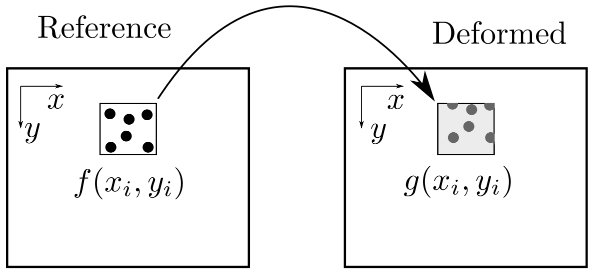

Figure 2. Schematic supporting the description of the displacement calculation procedure

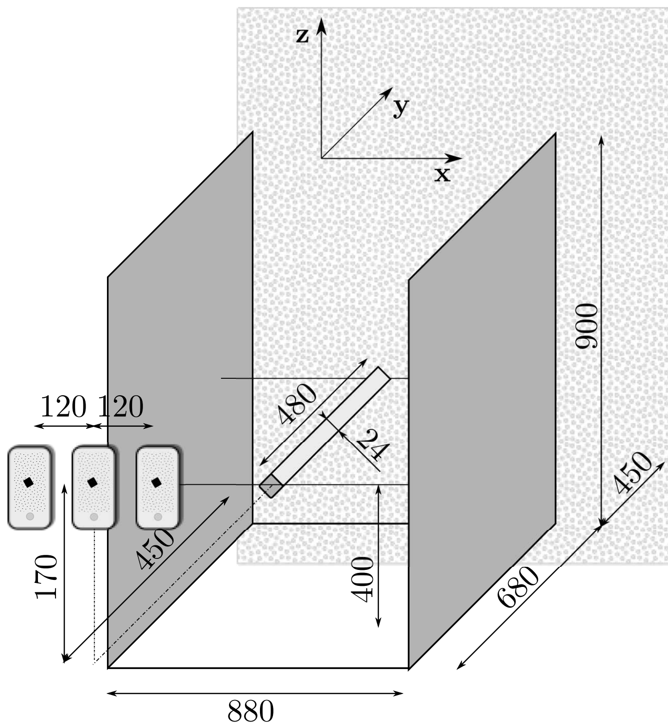

Figure 3. Sketch of the experimental setup with the smartphones configuration. Dimensions are in mm.

3. Experimental setup and procedures

An experiment was built to generate a flow that can be visualized with the BOS technique. The selected configuration consists of a heated horizontal square cylinder as the source of a thermal plume emerging from natural convection.

The heated body is an aluminium square tube of width $W = 24$ mm with an aspect ratio of 20 (480 mm). It is mounted such that the diagonals of the square section are vertical and horizontal as displayed in Fig. 3. It is held in place by two fine stainless steel wires at its two ends, and surrounded by two walls $18W$ apart from the cylinder axis. These wall are used to reduce the sensitivity of the convecting flow to the possible residual flow existing in the testing room of the laboratory.

The heating is operated by a custom system consisting of 20 silicon rectifier diodes operated with a constant current. This forms a 440 mm long heating element which is placed inside the heated

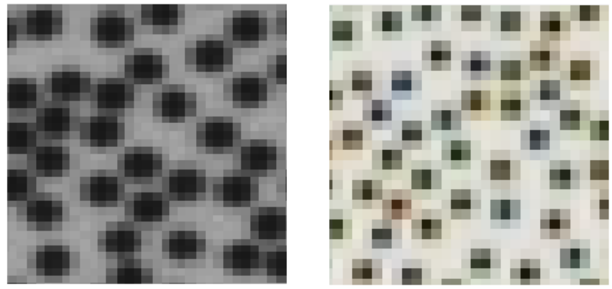

Figure 4. Images of the speckle obtained with the M5-lite 5M camera (Left), and the OPPO A74 camera (right).

cylinder. Both ends of the cylinder are then closed and it is filled with water to ensure a uniform heat distribution all along the tube.

A total power of 13 W is dissipated by the body at the experimental condition. The walls reach a temperature $T_w = 60°\text{C} \pm 2°\text{C}$, 40°C above the ambient temperature T_{amb}.

Part of this heat is radiated and the remaining is emitted by natural convection. If one assumes an emissivity of 0.05 for the aluminium tube, 0.7 W are radiated, which means that the natural convection coefficient is $h = 6.5\ \text{W/m}^2\text{/K}$. The Rayleigh number based on the cylinder cross-section width is $R_a = 7 \times 10^4$ implying that the boundary layer is laminar (Misumi et al., 2003). The thermal plume generated by this apparatus is consequently expected to be initially laminar and eventually transition to turbulence. The transition is expected to occur at a Grashof number based on the distance z from the axis of the cylinder

$$Gr_z = \frac{g\beta(T_w - T_{amb})z^3}{\nu^2} \tag{6}$$

in the range of $Gr_z = 2\times10^8$ to $Gr_z = 10^9$ (Noto et al., 1999), with $\beta = 1/T_{amb}$, $g = 9.81\ \text{m/s}^2$ and the viscosity $\nu = 1.610^{-5}\ \text{m}^2\text{/s}$. The Transition is thus expected to start at a distance of approximately 14 W above the cylinder.

For the purpose of assessing the BOS measurement systems, it was chosen to focus on this region of transition, however the signal intensity strongly decreases as soon as mixing occurs and thus gradients in refractive index become smaller. As a consequence, the cameras were placed in an intermediate position of $z = 11\ W$ which is a region where the flow strongly sways but is not yet fully transitional.

Two camera configurations are presented hereafter. The first is based on a LaVision M5-lite 5M device with a resolution of 2448x2048 pixels. It is equipped with a 12 mm lens (f/11) and controlled within the standard LaVision framework. The optical axis of the lens is set parallel to the cylinder axis with an offset in z of $11\ W$.

The second uses three OPPO A94 cameras. They are set with a resolution of 1920x1080 pixels (full HD) and operated by the free application *Open Camera* which gives some control on the camera settings such as the ability of disabling most of the image enhancement processing and forcing the video compression to be low.

The central camera is put in place of the LaVision M5-lite 5M and is accompanied with two cameras at the same vertical position z but offset in x by $+5W$ and $-5W$. *Open Camera* allows triggering the recording with a noise, all the tree camera are therefore roughly synchronised by triggering with a clapping sound. In addition a laser which dot is visible from all the three cameras is turned off soon after all camera are triggered. This allows to refine the synchronisation by tracking the extinction of the dot image per image. This configuration therefore results in three videos synchronised with a precision of one image from three different points of view. This precision is not as good as that of scientific cameras triggered by a timing unit, and is probably too low to use smartphones for a tomographic reconstruction without further improving the synchronisation procedure.

In both configurations the exposure time is set to 1/100 s, and the frame rate is 30 fps which is a value that is constrained by the smartphone specifications. The phones export videos in *.mp4* format, with h-264. The compression is set to be as low as permitted by the phone, which lead to videos of approximately 700 MB per minute.

The effective focal length of the smartphone cameras is adjusted to fit approximately that of the M5-lite 5M by using a built-in numerical zoom of $\times 1.19$.

Two different speckles were used to obtain dots covering approximately 3 to 6 pixels on the camera sensors. Examples of the speckle images are presented in Fig. 4. The speckle filmed with the LaVision camera was made of dot of diameter 1.6 mm, whereas dots of 2 mm are used with the smartphones. The diameter of the dots filmed by the LaVsion camera was close to 8 pixels, which was slightly larger than the optimal diameter for BOS processing.

4. Performance analysis of comBOS

A first series of 900 images (30 s) was recorded with the LaVision M5-lite camera and are processed both with the commercial software DaVis 10 and with comBOS. The subset size is 17 pixels in each case and no overlap is used.

To be processed by comBOS, images which were recorded via the dedicated software Davis 10 must first be converted from their native format 12 bit (*.im7*) to the standard 8 bit bitmap (*.bmb*). This conversion may have an adverse effect of the image quality which would affect the displacement maps. This possible effect was assessed by converting back *.bmb* to *.im7* and comparing the displacement maps computed with Davis 10 from both the original data set and the one that was temporarily converted to a 8 bit bitmap. No significant difference was visible, thus both the *.bmb* and *.im7* images are assumed to be identical in the following.

Figure 5 displays the horizontal and vertical displacements calculated form the LaVision M5-lite camera with Davis 10 (a,d), and comBOS (b,e) from the same original image. The displacement maps calculated by the two methods are ranging within ± 0.1 pixel and are qualitatively the same. A quantitative comparison is performed by analysing a profile of displacements along the line plotted in red onto the maps. The comparison is provided in Fig. (c) and (f). Although the agreement between the two displacement computation methods is excellent, profiles of displacements computed with Davis 10 are found slightly smoother than those computed with comBOS. This is also a general observation that the noise level is higher on displacement maps computed with comBOS than with Davis 10. This is analysed by a measuring the noise level in the region marked by a red square where no hydrodynamic disturbances are observed. The noise figures are provided in Tab. 1 and confirm a 33% higher level of noise with comBOS. From the author's experience, this noise level is approximately two times larger than what can be obtained in ideal conditions with Davis or comBOS, probably because of the dot diameter that is slightly too large (see Fig. 4).

Table 1 also provides figures about the processing time for displacement calculation which were both performed on the same computer. Davis processing is approximately two times faster than comBOS processing. This difference is noticeable, but in both cases the computational time remains quite short, and processing thousand images can be done within the duration of a coffee break. Besides, one could expect that translating comBOS from Matlab to a compiled programming language would significantly increase its performances.

5. Performance of smartphone cameras

The ability of smartphone cameras to record images that can be used for BOS is now investigated. The experimental configuration is that presented in Fig. 3 with three OPPO A94. Examples of displacement maps computed with comBOS are presented in Fig. 6. For the comparison Fig. 5(b) is also included in Fig. 6(d). Figures 6(b) and (d) that are expected to show the same flow but at different instant compare well with each other. There is no doubt from these images that either the LaVision M5-lite camera or that of the smartphones are able to capture the same overall flow features. The noise level in maps computed form the smartphone images is even smaller than that of M5-Lite images (see Tab.1). This difference is mostly the result of a better suited speckle for the smartphone than the scientific camera, but it denotes that the smartphone camera is capable of recording images for BOS processing even with video compression.

Even though the effect of video compression is not visible on snapshots the previous analyses do not prove that displacement maps are consistent in time. The mpeg video compression is indeed both operating in space and in time. The compression in time could cause an update of displacements by steps every 16 images because of the algorithm used by non-scientific camera. This facet is examined by comparing the power density spectrum $S_{\delta\delta}$ of the displacements at the same location marked by a red dot in Fig. 6(b) and (d) computed for both the smartphone and the scientific camera 30 s (900 images) records. Displacements were first converted to deflection angle δ with

$$\delta = \frac{\Delta_x d_{\mathrm{pix}}}{l_0} \tag{7}$$

where d_{pix} is the size of a pixel of the camera projected onto the background, and l_0 is the distance between the camera sensor and the the point of interest on the speckle. The steps in time of the

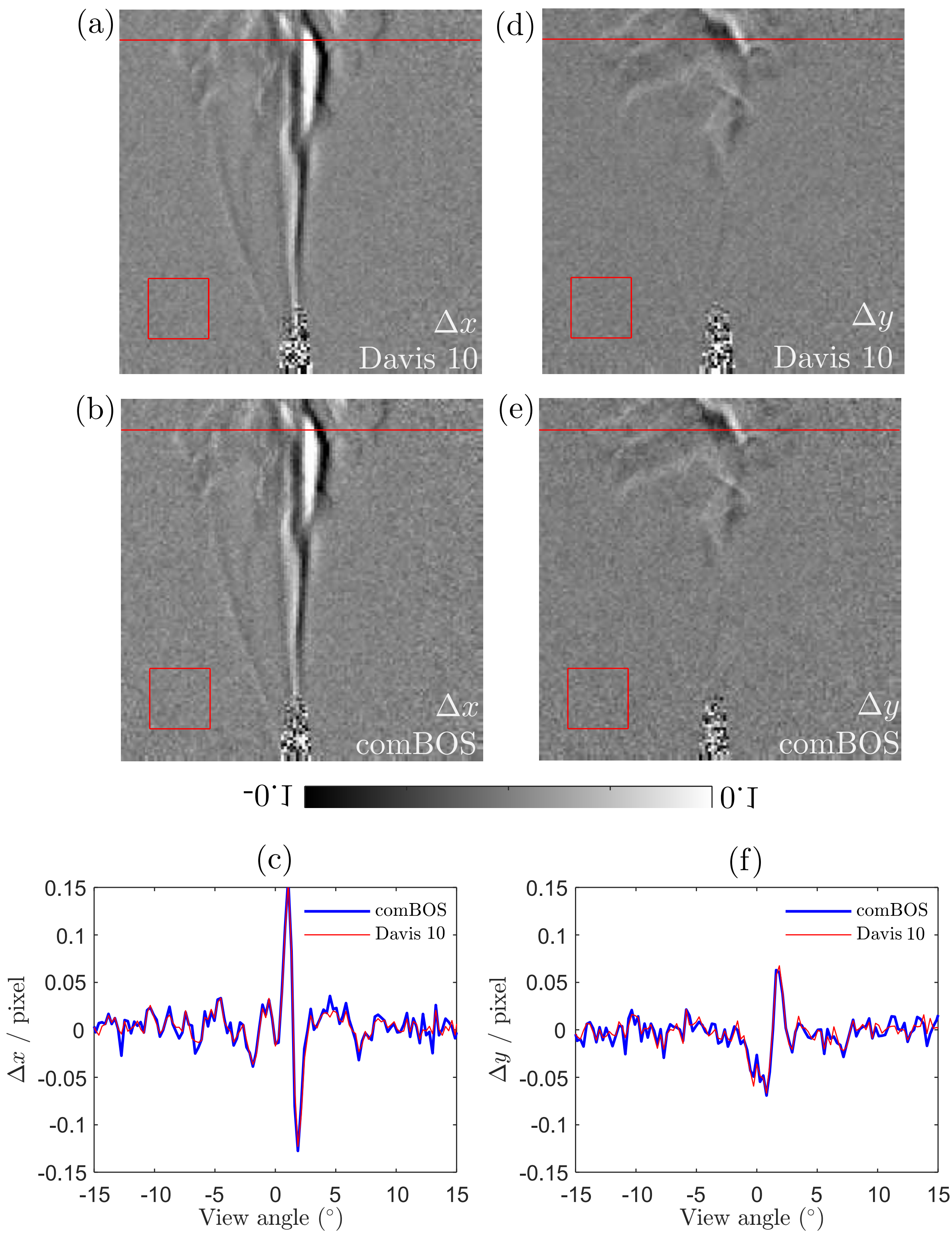

Figure 5. Comparison of the displacements Δx (a,b,c) and Δy (d,e,f) calculated from the M5-lite camera with Davis 10 (a,d) and the Matlab application (b,e). The red line is the probe for the profiles of displacements (c,f). The square shows the region for noise standard deviation calculations.

	M5 Lite	OPPO A94
$< \Delta x'^2 >^{1/2} \times 10^{-3}$ pix. - Davis 10	6.1	
$< \Delta x'^2 >^{1/2} \times 10^{-3}$ pix. - comBOS	7.8	4.5
$< \Delta y'^2 >^{1/2} \times 10^{-3}$ pix. - Davis 10	6.2	
$< \Delta y'^2 >^{1/2} \times 10^{-3}$ pix. - comBOS	8.3	4.6
Time (s) / im - Davis 10	0.4	
Time (s) / im - comBOS	0.9	1.2

Table 1. Summary of noise statistics for the considered configuration, and computing time of displacements.

displacements cause by the compression would induce a deficit of power at intermediate frequencies and an overestimation at higher frequencies. Spectra are shown in Fig. 7 and agree well with each other. This indicates that compression in time does not have any significant impact on the time evolution of the displacement maps.

In this configuration, we used three smarthpones to film the flow from three different angles. This is not enough to perform a 3D reconstruction of the flow with tomography algorithm, but it already has some interesting outcomes. First, Fig. 6(a) and (c) demonstrate the three-dimensional nature of the flow, which was not visible on Fig. 6(a) with the center camera. It also serves the educationnal approach by recalling to the user that the BOS integrates the flow over the optical path between the camera and the background. This integration makes the interpretation of BOS displacement maps difficult. For instance, it is not possible to determine the origin of the isolated structure marked with blue arrows in Fig. 6 with a single camera. This structure could be generated by an object that is not directly related with the heated cylinder such as a power supply cable or a light source. This ambiguity can be raised because the structure can be identified at a respective angles of 4.5° and -11° in Figs. 6 (a) and (b). The position of the structure is then determined by triangulation and is found to be at a distance of approximately 470 mm from the camera, which is close to one tip of the heated cylinder. This structure is actually a vortical flow emerging at the end of the cylinder that slowly oscillates from left to right.

5.1. Database of suitable smartphones

The present study focuses on the OPPO A94 camera, nevertheless, this experimental setup has also been used as a benchmark for other Android phones. Most of them were well suited for BOS applications, only a few cannot be used because images suffered from too much of processing or compression. The result is displacement maps of uniform patches that are not consistently updated in time. A list of the tested phones is provided in Tab. 2.

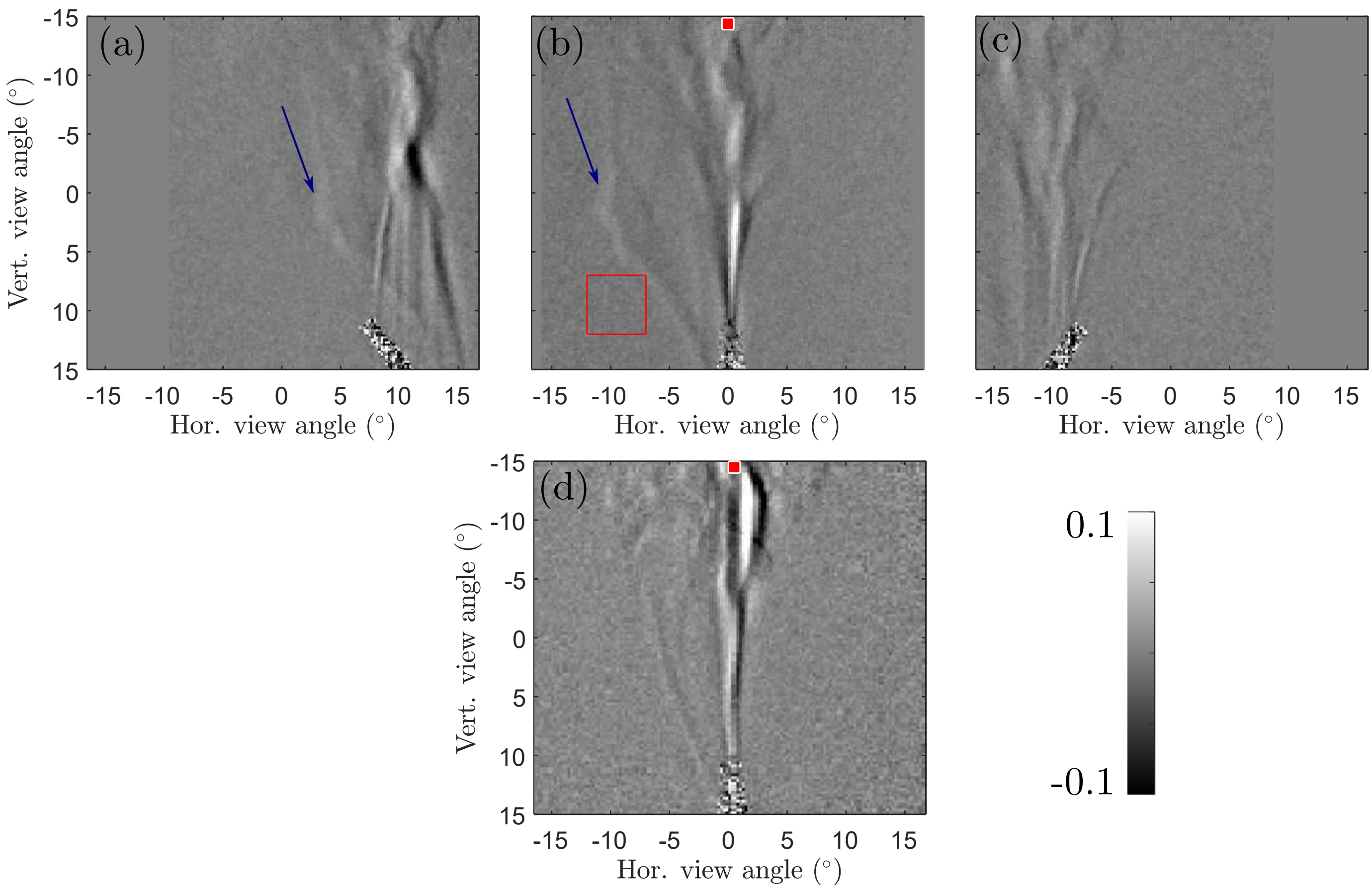

Figure 6. Displacement maps Δx obtained synchronously from the three OPOO A94 cameras (a,b,c) and with the M5-lite camera at a different instant (d) . The red square shows the region for noise calculations, the red dot is the position selected for computing the spectrum of the displacements and the blue arrows mark a feature of interest.

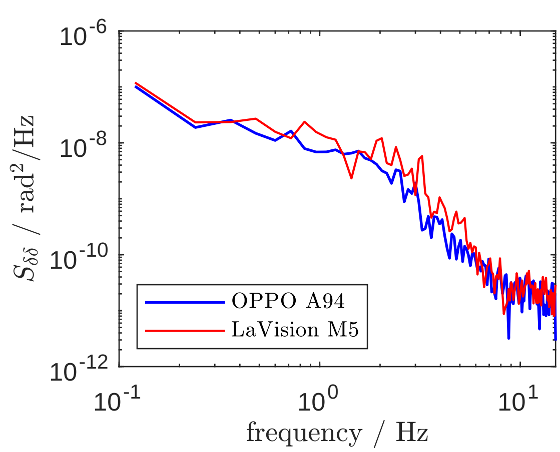

Figure 7. Power density spectrum of the BOS deflection at the location marked in Fig. 6.

Suitable for BOS	Not Suitable for BOS	Cost /€
Huawei P20 lite	Xiaomi RedMi 8	150
Xiaomi redmi 10		
Samsung Galaxy S9		
OPPO A94		
OPPO Reno 2z		
Samsung A50	Pixek 4A	300
Samsung Galaxy A52	OnePlus Nord	
Samsung Galaxy S10		
Samsung Galaxy S20 FE		
Samsung Note 10+	OnePlus 7t	600

Table 2. List of phones tested with the present experimental set-up sorted by cost from the cheapest to the most expensive.

6. Ongoing work and conclusions

Work has been conducted to assess the possibility of carrying out Background Oriented Schlieren experiments with a limited cost. For that purpose, a demonstration of the performances of the combination of a smartphone camera and the recently released comBOS Matlab toolbox is proposed with the visualization of the thermal plume over an heated square cylinder by natural convection.

A quantitative analysis demonstrates the ability of comBOS to compute displacement maps that are almost identical to that obtained with a commercial apparatus based on a LaVision M5-Lite camera and Davis 10. Only the noise level is found 33 % higher with the open source solution.

Images obtained from the camera of an OPPO A94 smartphones were processed with comBOS. The control of the camera with the third party android app Open Camera allows to record images with a minimum of preprocessing within the phone and a low level of compression. The resulting displacement maps are as good as with the scientific camera which exceeds our expectations. Similar results were found with most of the tested Android Smartphones which proves the possibility of performing non-expensive BOS in an educational context. The drawback of this configuration is the lack of flexibility on the camera settings such as the frame rate or the inaccuracy of the trigger. These limitations may be critical for some research applications, for instance to capture high speed phenomena.

7. Acknowledgement

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