

Background Oriented Schlieren technique with reconstruction of three-dimensional vector fields for pressure measurement of laser induced underwater shock waves

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Keywords: Background-oriented schlieren (BOS) technique, three-dimensional vector field, noncontact measurement, pressure field, underwater shock wave

ABSTRACT

We propose a novel reconstruction of the three-dimensional vector field (VR) of an axisymmetric target to overcome the problems that existing background-oriented schlieren (BOS) techniques based on reconstruction of three-dimensional scalar field (SR-BOS). This approach is based on an approximate relation between the projection of the axisymmetric vector field and the reconstructed vector field. The remarkable feature of the proposed technique is the reconstruction of an axisymmetric vector field with nonzero divergence, such as the field of a laser-induced underwater shock wave. Reconstruction of 3D vector fields, such as conventional vector tomography, is usually used in the field of electromagnetics with vector fields of zero divergence, and its application to fluid phenomena has been difficult. We have successfully applied this technique to BOS (VR-BOS) and measured the pressure fields of underwater shock waves. Using the experimental data and previous research data (Hayasaka et al. (2016)), the superiority of VR-BOS was clarified by comparing the measurement results of SR-BOS and VR-BOS, for example, measurement accuracy, in better convergence, less dependence on the spatial resolution of the acquired images, and lower computational cost. In particular, the fact that the measurement accuracy of VR-BOS is almost independent of resolution is expected to expand the maximum pressure values that can be measured by the BOS technique (Shimazaki et al. (2022); Hayasaka et al. (2016)). We also used synthetic data to investigate the effect of noise on VR-BOS. If the synthetic displacement contains noise, the noise is amplified in the vicinity of the axisymmetry axis in the reconstruction distribution calculated by VT. If the distance from the axisymmetry axis is more than 1/4 of the measurement range, the measurement accuracy of the reconstructed distribution can be sufficiently guaranteed. This proposed technique can be applied fluid dynamical fields, as well as other axisymmetric targets with vector reconstruction fields in other areas, such as electromagnetics and material mechanism.

1. Introduction

A laser-induced underwater shock wave is used in medical engineering field for various treatments such as lithotripsy (Sankin et al. (2008)), skull base tumor removal (Ogawa et al. (2011)),

and pain relief after arthroscopic treatment of osteochondral lesions of talus (Zhang et al. (2020)). These treatments are advantageous because incisions are not necessary thus having lower risk for complications. However, these treatments have disadvantages that, for example, cavitation caused by shock wave would damage healthy cells and the influence of shock wave to human body is unknown. Thus the measurement of a pressure field of shock wave is crucial. However previous research uses point measurement systems such as a hydrophone, which disturbs flow field or restricts experimental condition (Herbert et al. (2006)). Therefore we direct our focus on a non-contact density field measurement system, i.e., Background Oriented Schlieren technique, which has been successfully used to visualize the pressure field of underwater shock wave (Yamamoto et al. (2015); Hayasaka et al. (2016)). Nevertheless, the measurable pressure field of BOS technique is around 1-2MPa, which is lower than 30 MPa, a threshold of cavitation in tissue-mimicking gelatin gel (Oguri & Ando (2018)). Thus, we aimed to increase the maximum measurable pressure value of BOS technique by improving the calculation process. In previous study, the BOS technique was combined with reconstruction of three-dimensional scalar field (we name this BOS with the scalar reconstruction as SR-BOS), which used a Poisson's equation. Since a Poisson's equation has to be solved by iteration, the iteration could not converge, and the accuracy of the calculated density and pressure fields heavily depends on the spatial resolution. Hence, in this research, we propose a reconstruction method of three-dimensional vector field that does not use a Poisson's equation, and applied it to the BOS technique (we name this BOS with the vector reconstruction as VR-BOS). In this abstract, we report the results of comparison between the accuracy of measured pressure field, convergence, spatial resolution, and computational cost of VR-BOS with those of SR-BOS, which was performed using experimental data.

2. Background Oriented Schlieren technique

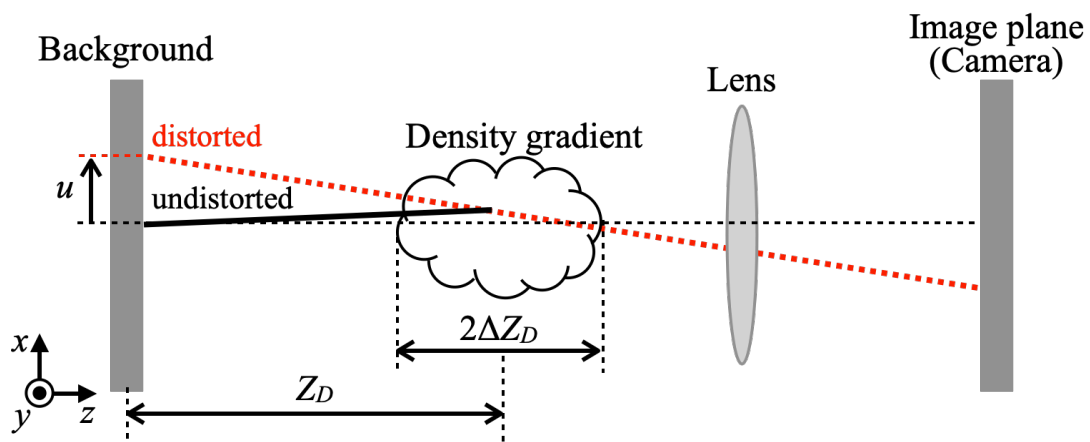


Figure 1. Principle of BOS technique.

The principle of the BOS technique is illustrated in Fig. 1. The background and the object are placed along the optical axis as shown in Fig. 1, and the pressure field is calculated from the captured

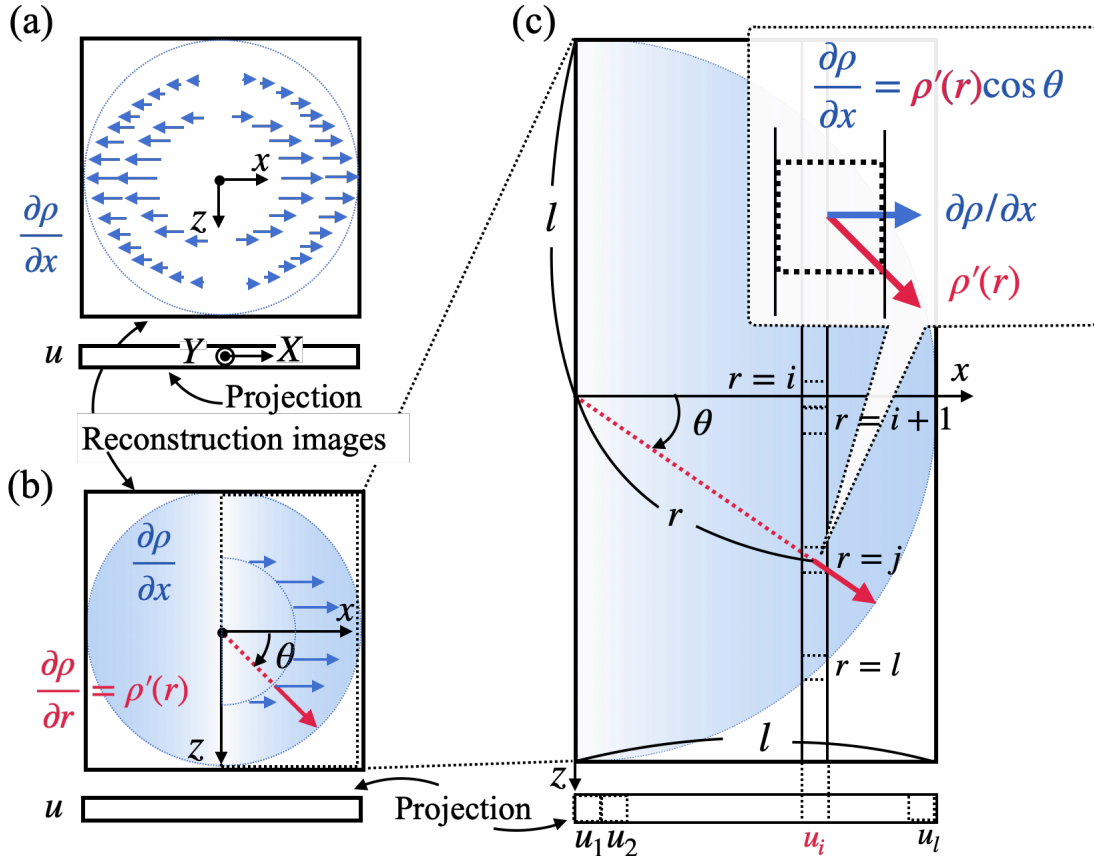


Figure 2. (a) The blue vector is the partial derivative of function ρ with respect to x . (b) The relation between $\partial\rho/\partial x$ and $\partial\rho/\partial r$. (c) Zoom up image of (b).

images. When the refractive index of a fluid changes, an apparent displacement (vector field) appears on the background. Displacement of the captured images in x -direction u is proportional to the density gradient $\partial\rho/\partial x$ integrated along the optical axis and can be expressed as follows (Venkatakrishnan & Meier (2004)):

$$u = \frac{Z_D}{n_0} \int_{Z_D - \Delta Z_D}^{Z_D + \Delta Z_D} \frac{\partial n}{\partial x} dz = \frac{K Z_D}{n_0} \int \frac{\partial \rho}{\partial x} dz, \quad (1)$$

where n is the refractive index of measurement target, n_0 is the refractive index of surrounding fluid, Z_D is the distance from the center of the measurement target to the background image, ΔZ_D is the thickness of the density gradient, and K is the Gladstone-Dale constant. Gladstone-Dale relation is established between refractive index and density,

$$n = \rho K + 1, \quad (2)$$

In the proposed BOS technique, VR-BOS, the density gradient field as shown in Fig. 2 is obtained by reconstruction of the three-dimensional vector field using the axisymmetric assumption. Using the axisymmetric assumption and applying the polar coordinate system to Eq. 1, we obtain

$$u \propto \int \rho'(r) \frac{x}{r} dz = \int \rho' \left(\frac{x}{\cos \theta} \right) \cdot \frac{x}{\cos \theta} d\theta, \quad (3)$$

where $x = r \cos \theta$, $z = r \sin \theta$, and $\partial \rho / \partial r$ is the density gradient in the radial direction. As shown in Fig. 2(c), the displacement u_i at position $x = i$ is discretely denoted using the density gradient in the radial direction $\rho'(r)$ and coefficient $\alpha(i, r)$ as

$$[u] \propto \frac{K Z_D}{n_0} [\alpha] [\rho'], \quad (4)$$

where $[\alpha]$ is the coefficients matrix and $[\rho']$ is the density gradient matrix. By solving Eq. 4, the density gradient in the radial direction $[\rho']$ can be obtained. Spatial integration is then performed on the density gradient field to calculate the density field. The pressure is calculated from the density using Tait equation (Vogel et al. (1996)),

$$\frac{p + B}{p_0 + B} = \frac{\rho}{\rho_0}^\alpha. \quad (5)$$

On the other hand, in SR-BOS, reconstruction of three-dimensional scalar field (SR) is applied to the divergence of the displacement, u' and v' which are the displacement in x -direction and y -direction, respectively.

$$\frac{\partial^2 \rho}{\partial x^2} + \frac{\partial^2 \rho}{\partial y^2} = \frac{K Z_D}{n_0} [\alpha] [\rho'], \quad (6)$$

This Poisson's equation (Eq. 6) is solved by iteration to obtain the density field using Gauss-Seidel method.

3. Experiment

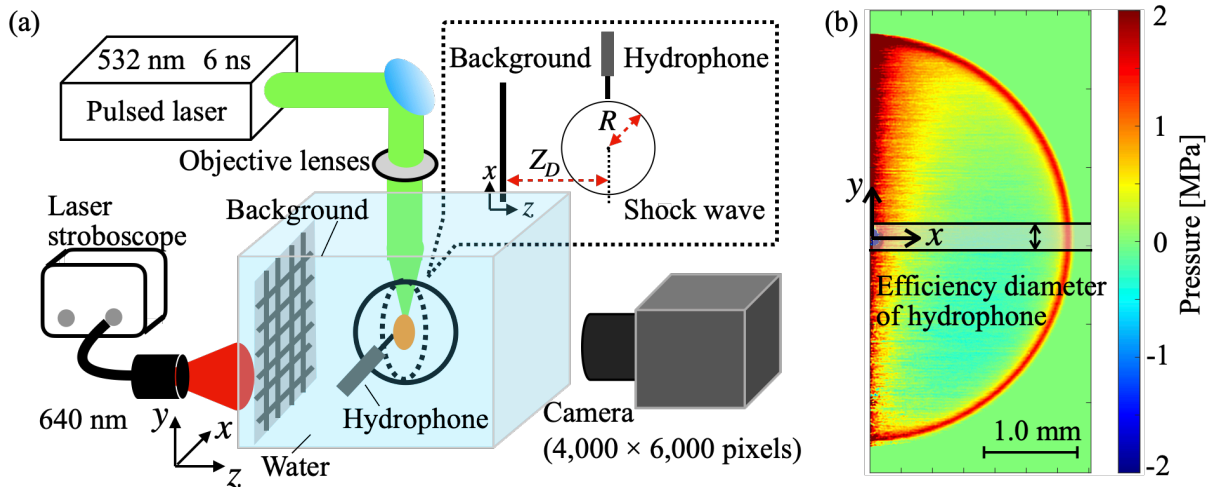


Figure 3. (a) Experimental setup. (b) Pressure field measured by VR-BOS. The effective diameter of the hydrophone is shown by the gray area.

Fig.3(a) shows the schematic of experimental setup. A pulsed laser (Nd: YAG laser Nano S PIV, Litron Lasers Ltd., wavelength: 532 nm, pulse width: 6 ns) is focused through an objective lens

(SLMPLN 20X, Olympus, magnification: 20 times, NA value: 0.25) onto a point inside a container filled with ultrapure water to generate an underwater shock wave. The background with checkered pattern is placed 1.5 - 2.0 mm behind the center of the shock wave. A high-resolution camera (EOS 80D, Canon, $4,000 \times 6,000$ pixels) captures the background images with and without underwater shock wave. At the same time, the shock wave pressure is measured by a hydrophone which has an effective diameter ± 0.25 mm from its center (Müller-Platte Needle Probe, Müller Instruments, Germany). The light source is a pulsed laser (SI-LUX 640, specialized Imaging Ltd., wavelength: 640 nm, pulse width: 10 - 20 ns).

Fast Checkerboard Demodulation (FCD) is employed for detecting the apparent displacement (Wildeman (2018); Shimazaki et al. (2022)). We compared the pressure measured by the hydrophone, SR-BOS, and VR-BOS using the experimental data of Hayasaka et al. (2016) (Experimental value (i)) and our experiment (Experimental value (ii)). The pressure field measured by BOS technique as shown in Fig. 3(b) was averaged in the y-direction over the effective diameter of the hydrophone for comparison with the pressure measured by hydrophone.

4. Experimental results

4.1. measurement accuracy

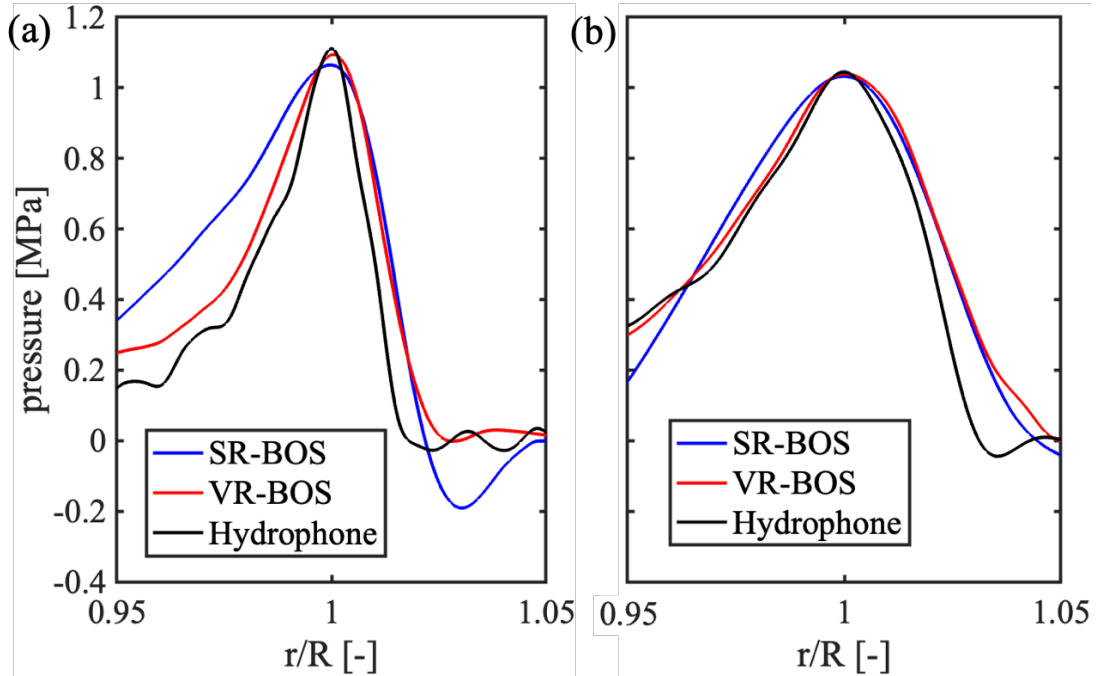


Figure 4. Pressure of the shock wave. R is the diameter of the shock wave. The data measured in (a) previous study (Hayasaka et al. (2016)) and (b) current study.

Fig.4 shows the pressure measured by hydrophone, SR-BOS, and VR-BOS. The accuracy of pressure measurement is compared for the range of $0.95 \leq r/R \leq 1.05$. For both Experimental values

(i) and (ii), the pressure curve measured by hydrophone is closer to that by VR-BOS than that by SR-BOS.

To quantitatively compare the accuracy of pressure calculations, we use the ratio S ,

$$s = \frac{S_h(r) - S_B(r)}{S_h(r)}. \quad (7)$$

Where the area S_B is the difference between the pressure calculated by the BOS technique and the measured pressure, and the area S_h is the difference between the measured pressure and 0. However, the closer the ratio is to 1, the closer the hydrophone pressure value is to the pressure value of the BOS technique, and the more accurate the pressure calculation is.

Table 1. Ratio S of the pressure measurements using SR-BOS and VR-BOS.

	SR-BOS[%]	VR-BOS[%]
Experimental value (i)	1.17	1.70
Experimental value (ii)	1.14	1.27

From Table 1, for both Experimental value (i) and (ii), the measurement error of VR-BOS is lower than that of SR-BOS. Therefore, the pressure measurement accuracy of VR-BOS is higher than that of SR-BOS.

4.2. convergence

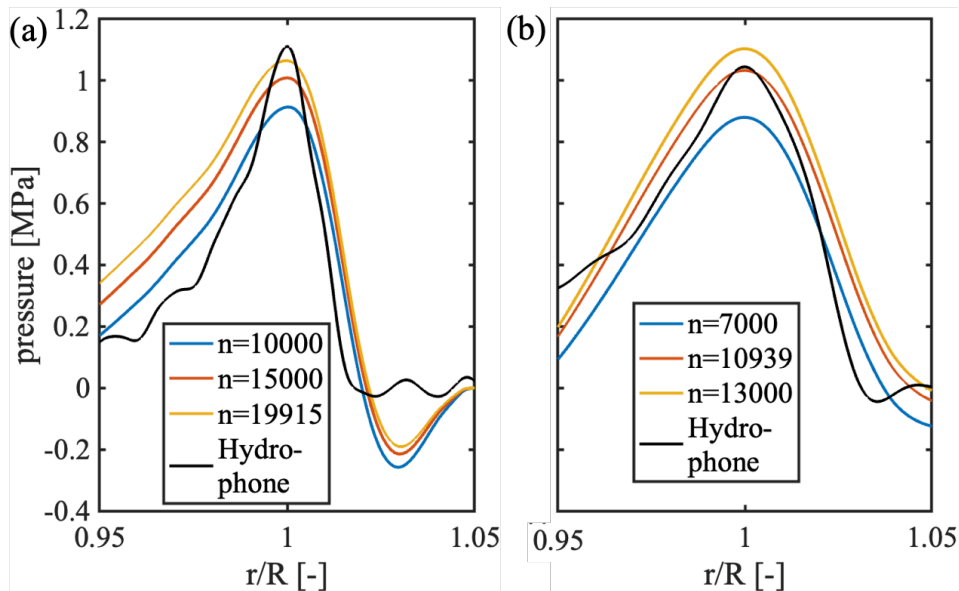


Figure 5. Calculated pressure of the shock wave after n iterations for (a) Experimental value (i) (Hayasaka et al. (2016)) and (b) Experimental value (ii).

Poisson equation of Eq. 6 is necessary to be solved by iterative calculation for calculating pressure in SR-BOS. Hayasaka et al. (2016) defined the iterative condition as

$$E(n) = \frac{\rho_{i,j}^n - \rho_{i,j}^{n-1}}{\rho_{max}^n}, \leq E_0. \quad (8)$$

where (i, j) represents the position in a Cartesian coordinate system, ρ_{max}^n is the maximum density value after n iterations, and E_0 is 1.0×10^{-8} which is defined by Hayasaka et al. (2016). When $E(n)$ is less than or equal to E_0 , the iteration is considered to have reached the convergence. For experimental value (i), the number of iterations satisfying Eq. 8 is $n = 19,915$. Fig. 5(a) and Fig. 5(b) show the results of the calculated pressure after n iterations until Eq. 8 is satisfied for experimental value (i) and experimental value (ii), respectively. 8(b), the iteration continued even when the calculated pressure curve is very similar to that of the hydrophone when $n = 10,939$. This is because, when $n = 10,939$, $E = 7.3 \times 10^{-8}$, which is larger than E_0 . On the other hand, VR-BOS can uniquely determine the pressure from Eq. 4 and Eq. 5 and it is not necessary to define E_0 .

4.3. Spatial resolution and computational cost

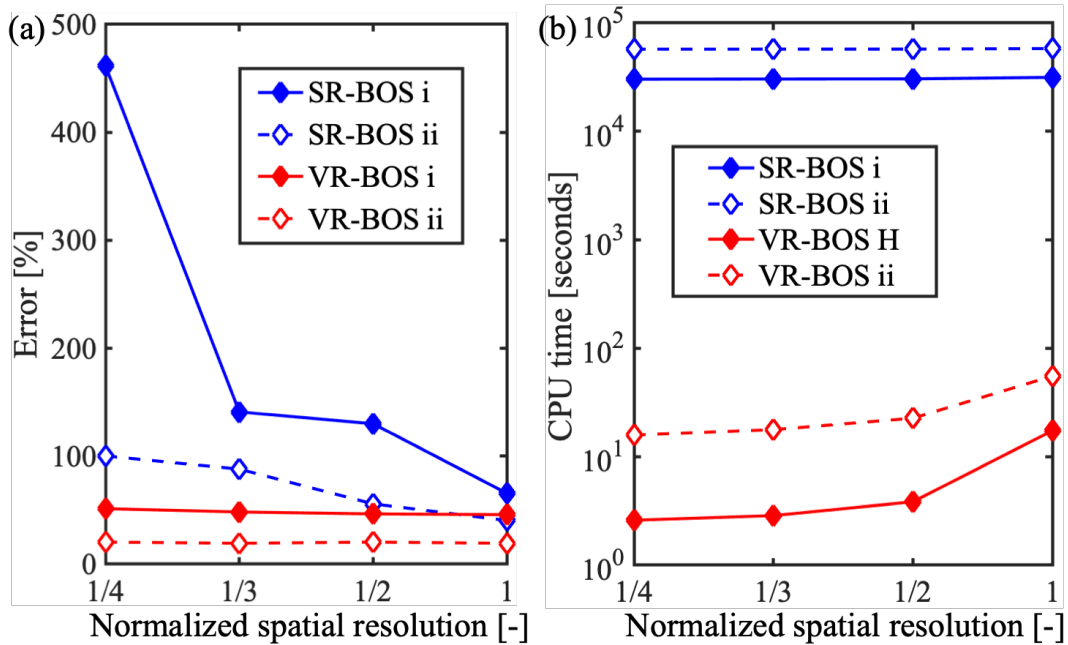


Figure 6. (a) Measurement error S and (b) CPU time of SR-BOS and VR-BOS for 1 (=original), 1/2, 1/3, 1/4 of the original spatial resolution of the displacement field.

To investigate the effect of spatial resolution on measurement accuracy, the spatial resolution of the displacement field is reduced before the pressure field is calculated using SR-BOS and VR-BOS. Fig. 6(a) shows the results of the measurement error of the pressure calculated by SR-BOS or VR-BOS with reduced spatial resolution. Since the spatial resolution of the displacement field is different between experimental value (i) and (ii), the normalized spatial resolution is used for

comparison. The original resolution is normalized as 1, and is reduced to 1/2, 1/3, and 1/4. As shown in Fig. 6(a), as the resolution decreases, the measurement accuracy of SR-BOS decreases for both experimental values (i) and (ii). In contrast, the change in the measurement accuracy with respect to resolution, is negligibly small for VR-BOS compared to SR-BOS.

Fig. 6(b) shows the computational cost of SR-BOS and VR-BOS for each resolution for both experimental value (i) and (ii), using the central processing unit (CPU) time. The CPU time starts from the reading of the data of the displacement field and ends when the calculation of the pressure field has been completed. There is an order of 103 s difference between the CPU time of VR-BOS and that of SR-BOS as shown in Fig. 6(b). Furthermore, the CPU time of VR-BOS becomes shorter with the reduction of the resolution.

5. Numerical approach: effective measurement range

5.1. Numerical condition

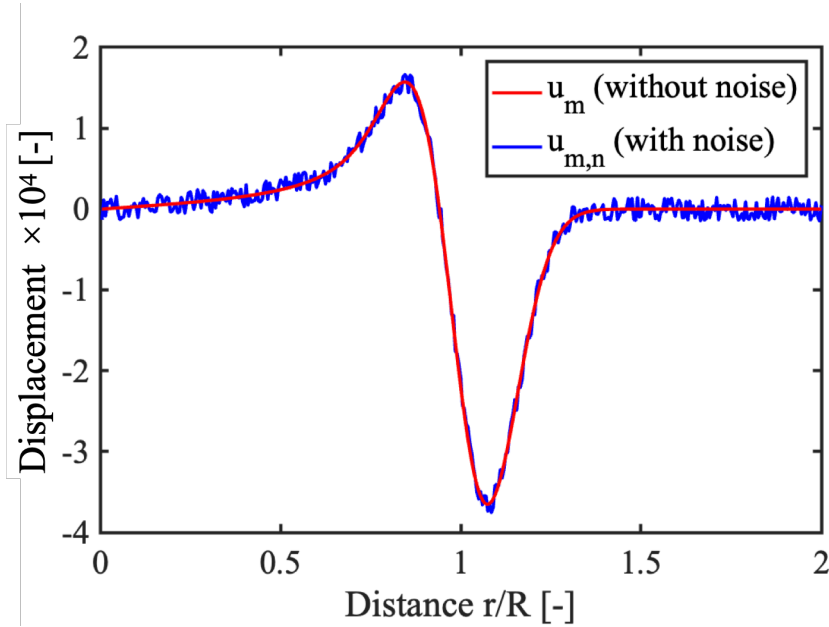


Figure 7. Distribution of synthetic displacement with and without noise in r/R direction. R is constant value, 100.

To investigate the effective measurement range of VR, synthetic data that can control noise is used. Let the synthetic radial r density without noise be a Gaussian distribution, denoted by

$$\rho_m(r) = \frac{1}{\sqrt{2\pi}a} \exp \left\{ -\frac{(r-b)^2}{2a^2} \right\}, \quad (9)$$

where a and b are constant 10, 100, respectively. Eq. 9 is differentiated in the r direction to obtain

a synthetic radial r density gradient field without noise,

$$\rho'_m(r) = \frac{r-b}{\sqrt{2\pi}a^3} \exp \left\{ -\frac{(r-b)^2}{2a^2} \right\}. \quad (10)$$

The density gradient field in the r direction distributed over the xz coordinate system is integrated in the z direction to create the synthetic displacements $u_m(r)$. All synthetic data are assumed to be dimensionless.

Noise is added to the synthetic displacement without noise u_m using the rand function in MATLAB. Fig. 7 shows the synthetic displacement without and with noise. Apply VT-BOS to the displacements u_m without noise and $u_{m,n}$ with noise, and calculate the density fields ρ_{vt} and $\rho_{vt,n}$. By comparing the synthetic density field ρ_m with the density fields ρ_{vt} and $\rho_{vt,n}$ calculated by VT-BOS, we clarify the differences between the cases with and without noise.

5.2. Numerical result

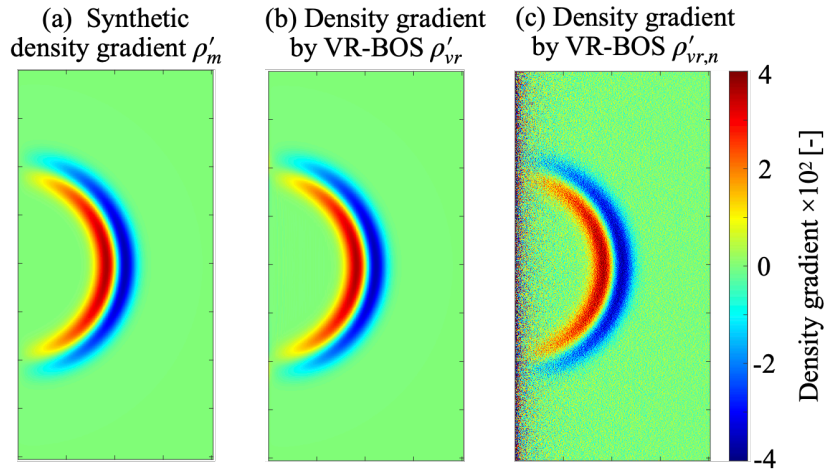


Figure 8. (a) Synthetic density index gradient ρ'_m . (b) Density index gradient ρ'_{vr} calculated from synthetic displacement without noise by VR-BOS. (c) Density index gradient calculated $\rho'_{vr,n}$ from synthetic displacement with noise by VR-BOS.

Fig. 8 shows the synthetic density gradient field ρ'_m , the density gradient field ρ'_{vr} calculated by applying VT-BOS to the synthetic displacement u_m without noise, and the density gradient field $\rho'_{vr,n}$ calculated by applying VT-BOS to the synthetic displacement with noise $u_{m,n}$. Both ρ_m and ρ_{vr} show nearly equal density fields, while the noise in $\rho_{vr,n}$ increases as one approaches the z -axis.

Fig. 9 plots the $x = 0$ data for each density gradient field in Fig. 8. The plots of ρ_m and ρ_{vt} are nearly identical in Fig. 8. The fundamental wave of $\rho_{vr,n}$ at $r/R > 0.5$ has a shape similar to that of ρ_m and ρ_{vt} . While, the noise of $\rho_{vt,n}$ for r/R less than 0.5 increases as it approaches the origin, and is not similar to the waveforms of ρ_m and ρ_{vt} .

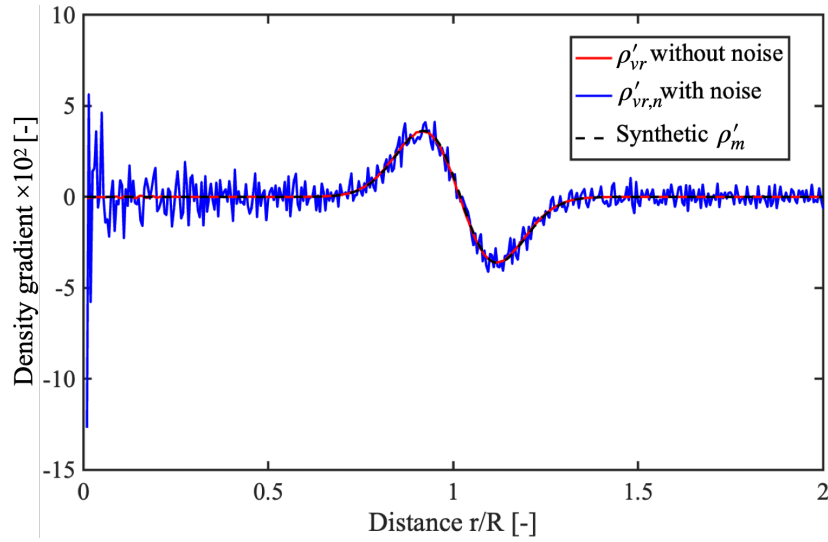


Figure 9. Distribution of the synthetic density gradients and density gradients with and without noise in the $x = 0$ direction on xz coordinate of Fig. 8

In the density gradient field calculated by VR-BOS, the cause of the noise increase near the z -axis lies in the theoretical formula of VR (Eq. 3). 3 is multiplied by $1/\cos\theta$. It is known that the smaller the value of θ , the larger the value of $1/\cos\theta$. Therefore, the closer to the axis of the axial object (z -axis), the more the noise is amplified, and the less reliable the values near the z -axis are in the pressure field calculation. Therefore, noise is amplified in the region closer to the z -axis of the axisymmetric object, and values near the z -axis are less reliable in the pressure field calculation.

6. Conclusion

We have newly developed a reconstruction method of three-dimensional vector field and applied it to the BOS technique (VR-BOS). By using experimental data of Hayasaka et al. (2016) and our own experiments, we compared VR-BOS with the conventional SR-BOS in terms of measurement accuracy, convergence, spatial resolution, and computational cost. As compared to SR-BOS, the pressure measurement by VR-BOS has a higher accuracy, the pressure can be uniquely determined, the accuracy is resolution-independent, and lower computational cost. From the synthetic data, it is clear that VT-BOS has an increase in noise near the central axis of the axisymmetric target. Therefore, VR-BOS is superior to SR-BOS in measuring axisymmetric three-dimensional pressure or density fields.

Acknowledgements

This work was funded by Japan Society for the Promotion of Science (Grant No. 20H00223, 20H00222, and 20K20972) and Japan Science and Technology Agency PRESTO (Grant No. JP-MJPR21O5).

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