

Aerosol structure measured by double field of view digital sideband holography

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ABSTRACT

Digital sideband in-line holography is a technique, based in classical digital in-line holography, that allows the measurement of the position and velocity of the particles in a three dimensional fluid flow. The great advantage of this technique is its very simple set-up, which basically requires a camera, a lens and a frequency filter to record the holograms.

The sideband configuration removes the twin image and prevents the recording of unwanted frequencies that causes aliasing, therefore increasing the Signal-to-Noise Ratio. In this work it has been applied for the quantitative characterization of a three-dimensional aerosol, where droplet velocity and position have been measured. The field of view that can be recorded with magnification $M \approx 1$ has been doubled by illuminating two volumes with the same beam that crosses the aerosol twice before reaching the camera sensor. These two fluid volumes are analyzed independently while keeping a high spatial resolution in the axial component.

1. Introduction

Digital holography (Schnars and Jüptner (2005), Arroyo and Hinsch (2008), Katz and Sheng (2010)) is a measurement technique that is being widely applied in many scientific and industrial fields. It is based on the recording in a digital sensor (CCD or CMOS camera) of the interference of light scattered by an illuminated object with a reference beam. The object information is retrieved mathematically. With digital holography it is possible to characterize fluid flows, obtaining the 3D particle position, velocity and size (Pu et al. (2005), Palero et al. (2007, 2010, 2013), Tian et al. (2010), Li et al. (2017), Hesselting et al. (2018)).

In digital in-line holography (DIH), a translucent object is usually illuminated with a collimated beam. The light scattered by the object forms the so-called object beam and the unscattered light,

forms the reference beam. This configuration presents several disadvantages. First, the twin image problem (Denis et al. (2008), Ooms et al. (2008), Lobera et al. (2020)), that implies that the object virtual and real images are not fully separated when the hologram is reconstructed. Besides, the twin image adds noise to the reconstruction. Second, the low system aperture, due to the sensor pixel size, makes the length of the reconstructed particle image along the observation direction several times bigger than its actual size. As a consequence, the accuracy in determining the position of the particle along the optical axis and the spatial resolution decrease. Finally, the flow cross-sectional dimension that can be recorded is limited by the size of the camera sensor, typically a few square centimeters. Despite these drawbacks, digital in-line holography is a powerful technique for measuring the three velocity components (3C) in a volume (3D) as well as particle and droplet sizes (Chen et al. (2017), Shao et al. (2019), Toloui et al. (2017), López et al. (2020), Gómez-Climente et al. (2022)).

Digital sideband holography (Bryngdahl and Lohmann (1968), Palero et al. (2014), Ramírez et al. (2015), López et al. (2024)) has been applied in this work. This technique presents several advantages. In the first place, the very simple optical set-up that only requires a camera, a lens and a frequency filter to record the holograms. In the second place, it solves the twin image problem and prevents the aliasing by cutting off the frequencies that are limited by the system aperture, i.e. by the sensor pixel size, and do not contribute to the signal.

With this technique we have measured the position and velocity of a dense spray while maintaining a high spatial resolution, as well as accuracy. The field of view that can be recorded by the sensor at a magnification $M \approx 1$ has been doubled, by illuminating two sections of the spray ($22 \times 22 \times 100 \text{ mm}^3$ each) with the same beam that crosses the fluid twice before reaching the camera sensor. One of the most remarkable features is that the volume longest dimension is parallel to the optical axis.

Hologram reconstruction and analysis has been carried out adapting a method described in (López et al. (2020, 2024)) to the present experiments. The strength of this method lies in a fully automatic, adaptive process that provides the three components of the particle position with very high accuracy. Particle velocity is obtained with a method called Adaptive Cross Correlation with Tracking From Beginning (ACCTFB), which combines the 3D cross-correlation of the reconstructed intensity for measuring the transversal velocity coordinates (V_x, V_y) and the 3D cross-correlation of the reconstructed object complex amplitude for measuring V_z .

2. Digital sideband holography

In digital sideband holography the particle field is imaged near the sensor plane with a lens, recording a so-called image hologram (figure 1a). However, in the lens focal plane, the particle field power spectrum and the illuminating beam, which appears as a bright spot in the center, can be considered as the object beam and the reference beam source of a lensless Fourier hologram respectively. The particle distribution power spectrum is duplicated in the hologram Fourier transform, since the recorded interference pattern is a real function. Then, it is possible to record only one half of the spectrum with an aperture (while blocking the other half) without losing meaningful information (Lobera et al. (2020)). So, a lensless Fourier hologram of the unblocked power spectrum is produced. In the hologram reconstruction, by calculating the hologram Fourier transform, we will retrieve the real and virtual images of the recorded part of the power spectrum completely separated (figure 1b). Characteristic of the sideband configuration is the particle image that appears as a half-moon (figure 1a) (Palero et al. (2014)).

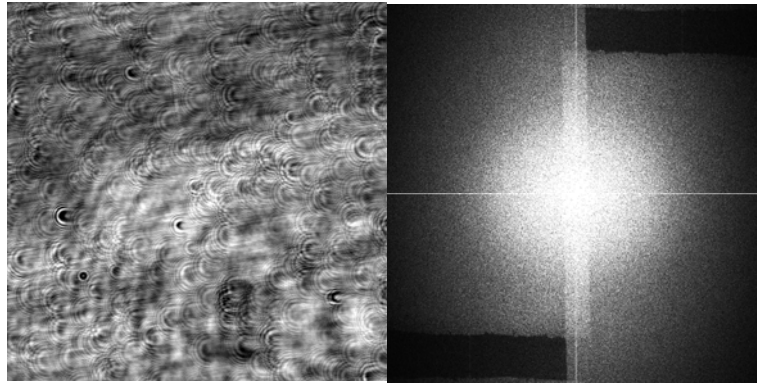


Figure 1: (left) Example of a sideband hologram. (right) Fourier transform of the hologram, where the real and virtual images of the aperture are separated. In this example the aperture has been misaligned on purpose.

In the hologram plane we record the interference between a reference beam $r(x,y)$ and the object beam $o_F(x,y)$, that has been filtered by the aperture located in the lens focal plane, and can be written as:

$$o_F(\mathbf{r}_0) = \int_{-\infty}^{+\infty} O(\mathbf{k})F(\mathbf{k})\exp(i\mathbf{k} \cdot \mathbf{r}_0) d\mathbf{k} \quad (1)$$

Where $\mathbf{k} = (k_x, k_y, k_z)$ is the wave vector and $F(\mathbf{k})$ is the aperture, i.e., the frequency filter. Thus, the intensity in the hologram plane is:

$$I(x, y) = |r(x, y)|^2 + |o_F(x, y)|^2 + o_F^*(x, y)r(x, y) + o_F(x, y)r^*(x, y) \quad (2)$$

In the spatial domain, these four terms are mixed, but they can be separated in the frequency domain by calculating the Fourier Transform of the recorded intensity. The two first terms of Eq. (2) will produce a continuous term that lies in the center. Meanwhile, the third and fourth terms will produce the real and virtual images of the filtered object, which are the most important terms:

$$\text{TF}\{I(x, y)\} \approx \text{TF}\{o_F^*(x, y)r(x, y) + o_F(x, y)r^*(x, y)\} \quad (3)$$

The filter that is placed on the Fourier plane is a 2D rectangular function. It can be as simple as a piece of black cardboard sheet where a rectangular aperture has been cut out. Its dimension is calculated considering the Nyquist criterion, that sets the maximum frequency of the recorded interference pattern as $f_{\max} = 1/(2d_{x,\text{sensor}})$, where $d_{x,\text{sensor}}$ is the sensor pixel size. The area in the Fourier plane with “recordable” information will be a centered square region whose size Δ depends on $f_{\max}$ such that $\Delta = 2\lambda D f_{\max}$, where λ is the wavelength and D the distance between the Fourier plane and the hologram sensor. When the image is formed on the sensor plane $D = Mf'$, M being the magnification and f' the lens focal length, and Δ can be calculated as:

$$\Delta = \frac{\lambda f' M}{d_{x,\text{sensor}}} \quad (4)$$

With a $\Delta \times \Delta$ size square mask placed in the lens focal plane, a classical DIH will be obtained. In the reconstruction, the twin images will appear overlapped, but this mask will effectively eliminate aliasing by cutting off the frequencies that do not contribute to the signal and only add noise. By using a rectangular mask a $(\Delta/2 \times \Delta)$ both aliasing and the twin image problem will be solved.

In the hologram reconstruction, one of the aperture images is selected, $O_F(\mathbf{k})$, and propagated by using the convolution approach (Goodman, (1996)). In that way, the object complex amplitude, $o_{1F}(x, y, z)$, is calculated in any plane by means of the inverse Fourier transform as:

$$o_{1F}(\mathbf{r}) = FT^{-1}\{O_F(\mathbf{k})H(\mathbf{k})\} \quad (5)$$

where, $H(\mathbf{k})$ is the propagation kernel which is given by:

$$H(\mathbf{k}) = \frac{\exp(ik \cdot |\mathbf{r} - \mathbf{r}_o|)}{ik \cdot |\mathbf{r} - \mathbf{r}_o|} \quad (6)$$

3. Experimental configuration and hologram recording considerations

In the following we will describe the experimental set-up (figure 2) designed to record a field of view twice the camera sensor size, while keeping $M \approx 1$. We used a squared-sectioned cuvette (100×100×100mm³), made of optical glass, to keep the spray confined. The illumination beam (continuum laser, $\lambda=532\text{nm}$.) is carried by an optical fiber, collimated and redirected by means of a pair of mirrors, so that it enters the cuvette through the wall closest to the camera (cuvette wall D). There, it travels to the cuvette opposite wall (C), where it is reflected back with a rectangular prism. In this second path, parallel to the first one, the beam travels towards the camera sensor from the plane B to A. In that way we can record two sections of the aerosol volume ($H \times W \times L = 22 \times 22 \times 100\text{mm}^3$ each), that are separated a certain distance in the X direction.

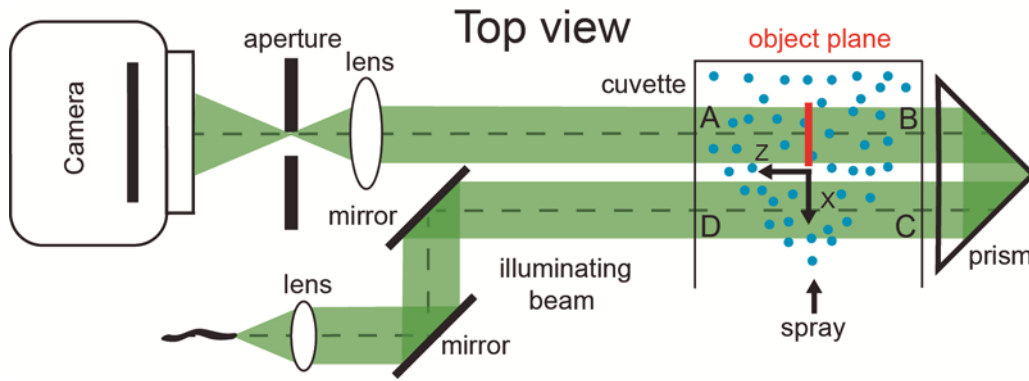


Figure 2: Experimental set-up. The illumination beam enters the cuvette at D towards C. After crossing the prism, enters back into the cuvette at B. The beam leaves at A towards the lens that forms the image of the center of the spray (object plane) on the camera sensor. In the lens focal plane, the rectangular aperture selects half of the power spectrum and removes the high frequencies.

As the illuminating beam travels through the prism and the cuvette, it crosses volumes with different refractive index which will increase the total optical path (OP). The cuvette walls have a width of 3.5mm and a refractive index of $n_{\text{glass}}=1.523$ (OP=5.33mm, each). The prism is 40×40mm, made of BK7 optical glass, $n_{\text{BK7}}=1.519$, and adds an OP=60.8mm. The total optical path added by these elements is 82.1mm, making a total path along the optical axis (TOP), in which we will find information about the particles, TOP=282.1mm. Therefore, the effective recorded volume is 22×22×282.1mm³. This volume is approximately centred in the X axis of the cuvette: AB goes from $-21\text{mm} < X \leq 0\text{mm}$ and CD from $9\text{mm} \leq X < 30\text{mm}$, and at $Y=17\text{mm}$ over the cuvette bottom.

The hologram plane (object plane in figure 2) was selected in the cuvette center, in the AB range, which coincides approximately with the axis of the spray. Series of holograms were recorded with a Photron SA2 camera (sensor size 2048×2048, 1000 fps) with a pixel size $d_{x,sensor}=10\mu\text{m}$. A photographic lens (MicroNikkor, $f'=105\text{mm}$) formed the volume image in the camera sensor with magnification $M=0.94$. As we reconstructed the holograms in the object domain, from now on we will use the effective pixel $d_x=d_{x,sensor}/M=10.6\mu\text{m}$. The size of the rectangular aperture in the lens focal plane was $5.25\times 2.625\text{mm}^2$.

4. Hologram reconstruction, droplet position and velocity measurement

We will distinguish two parts in the hologram analysis. The first one aims to determine the position of a set of droplets in the volume and the second one is the droplet tracking through a certain number of holograms. The analysis has been done by applying a procedure similar to the one described in (López et al. (2020, 2023)), adapted to the present case. Only the regions where there are spray droplets are being reconstructed. This means that the information corresponding to each section is independently extracted by reconstructing the hologram at different axial ranges. This entire process is completely automatic and is not affected by the droplet image shape.

In the reconstruction, the origin of the axial coordinate z is made to coincide with the hologram plane position. In particular, $z=0$ will be located in the center of the cuvette, which implies that some droplets in region AB are in $z<0$, while the rest is in $z>0$. Droplets in CD correspond to x -positive section, while droplets in AB belong to the x -negative volume. In an initial step, the holograms SNR are enhanced by subtracting an image of the reference beam and normalizing its intensity.

Let us start with the first hologram. The complex amplitude distribution of the object wave is reconstructed at different z -planes (with a sampling interval d_z) inside the volume of the cuvette. The measurement of the position of the droplets is based on the detection of intensity maxima in this reconstructed volume. The intensity $i_{1F}(x, y, z)$ is calculated from the object complex amplitude as:

$$i_{1F}(x, y, z) = |o_{1F}(x, y, z)|^2 \quad (7)$$

The volume is divided in 3D interrogation windows (3DIW), which size will decrease in successive steps in order to increase the accuracy of the droplet position measurement. In a first

step, these 3DIW will extend over the whole length of the reconstructed volume, with dimensions $40 \cdot d_x \times 40 \cdot d_y \times (2L/d_z)$ pixel³ (figure 3a), where $d_x=d_y=10.6\mu\text{m}$ and $d_z=2\text{mm}$. Thus, the first 3DIWs correspond to $0.424 \times 0.424 \times 200\text{mm}^3$. These 3DIWs were overlapped a 50% in the xy plane, providing around 10000 maxima in the first hologram. The spurious peaks and the maxima coming from oversampling due to the 3DIW overlapping are removed. The remaining maxima are divided in two sets according to its axial position, i.e., whether they belong to AB or CD.

We will start by analyzing the volume defined by AB. We compute again the object wave intensity, defining new and smaller 3DIWs centered in the maximum found previously (figure 3b). These 3DIWs are computed automatically, with an adaptive criterion that calculates the 3DIWs cross section ($S_x \times S_y$) as the size of the Airy disc of a circular aperture located at a distance, z_p , from the hologram plane. Therefore, the size of a given 3DIW is $(S_x/d_x) \times (S_y/d_y) \times 30\text{pixel}^3$ with $d_z=0.2\text{mm}$.

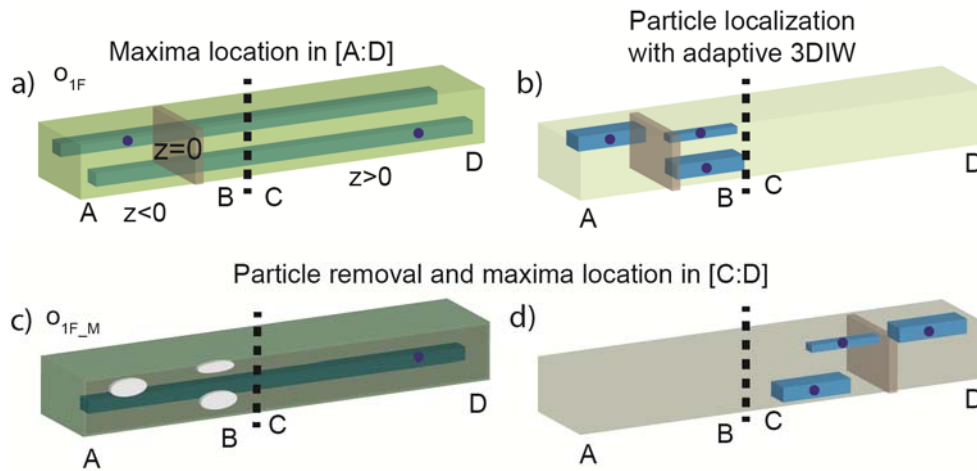


Figure 3: Steps in the process of locating the initial set of droplets. Intensity maxima are located in the reconstructed volume followed by droplet identification and accurate coordinate measurement in the AB range. Droplets located in this first volume are removed and the modified complex amplitude (o_{1F_M}) is used to detect droplets in CD.

Pixels in each 3DIW are ordered from highest to lowest intensity and the cumulative sum is calculated in a vector, in such a way that its last element is the total cumulative sum. The droplet will be defined by the elements whose intensity adds up the 64% of the total intensity in this 3DIW. The transversal coordinates (x, y) are given by the centroid of that final pixel distribution, while the droplet z coordinate is calculated as the maximum of a quadratic polynomial fit. The droplet is kept in the working set only if the new coordinates are close enough to the coarse maximum calculated previously, being the maximum shift allowed ($\Delta x < 0.025\text{mm}$, $\Delta y < 0.025\text{mm}$,

$\Delta z < 3\text{mm}$). Once the droplets in AB are located, a second step is performed to locate the maxima (figure 3c) with the modified complex amplitude, o_{1F_M} , where the intensity of the droplets located in AB has been removed. The 3DIW was again $40 \times 40 \times (2L/d_z)$ pixel³, with the same pixel sizes as before. Then, droplet localization, as described above, is carried out for the precise localization inside the CD volume ($z = 60\text{mm}$ to 160mm), where we reconstruct the complex amplitude and therefore, the intensity from the CD central plane instead of the hologram plane (figure 3d). The advantage of reconstructing the droplets from this central plane that we can define new and smaller 3DIWs centered in the maximum found previously, reducing the memory and calculation time requirements. After applying the algorithm restrictions to decide whether or not a location corresponds to a droplet, an average of 250-300 droplets per section is obtained.

Once we have defined the droplet set in the first hologram, these droplets are tracked along a number of holograms in order to obtain the 3D velocity field. We applied a multi-pass, adaptive interrogation process called Adaptive Cross-Correlation Tracking From Beginning (ACCTFB). This method is based on the 3D cross-correlation of 3D interrogation windows, in a similar approach to the one used in Particle Image Velocimetry (PIV). The process is outlined in figure 4.

Droplets in section AB have to be tracked first and removed from the hologram before tracking droplets in CD. The intensity is calculated and new 3DIWs ($80 \times 80 \times 15$ pixel³, $d_z = 0.2\text{ mm}$), are defined. This 3DIWs are centered in the droplet location in the first hologram. Two 3D cross-correlations are calculated consecutively, and on each of them, a Gaussian filter is applied. The first one aims to highlight the droplet against the background and is centered in the droplet position in the first hologram. Once the first cross-correlation is calculated, a second filter, centered in the position given by the correlation peak position, $\Delta \vec{r}_2$, is applied to the second hologram. Therefore, we can remove the contribution of out-of-focus droplets in the reconstructed volume. The intensity 3D cross-correlation is calculated again with an improved SNR.

$$C(\mathbf{r}_{3DIW}) = (i_{1F}(\mathbf{r}_{3DIW})G(\mathbf{r}_{3DIW})) \otimes (i_{2F}(\mathbf{r}_{3DIW})G(\mathbf{r}_{3DIW} - \Delta \mathbf{r}_2)) \quad (8)$$

The new correlation peak will provide the droplet displacement. Only the pixels whose intensity adds up the 64% of the total intensity are considered to form the peak. The transversal droplet displacement (Δx_2 , Δy_2) and the droplet velocity components ($V_{x2} = \Delta x_2 / \Delta T$; $V_{y2} = \Delta y_2 / \Delta T$) are calculated from the peak centroid. The droplet position in the second hologram (x_2 , y_2) is

obtained by adding to the displacements of the droplet coordinates in the first hologram (figure 4a).

In order to calculate the displacement along the optical axis (Δz_2) a third cross-correlation is needed (figure 4b). However, we use the complex amplitude instead of the intensity as shown in (López et al. (2020, 2023)). The complex amplitude cross-correlation uses the whole information of the droplet image, its amplitude and its phase, so it provides a sharper correlation peak than the intensity cross-correlation (Wormald and Coupland (2009), Coupland et al. (2000)). The final size of the 3DIWs is reduced to the droplet size ($8 \times 8 \times 15$ pixel³; $d_z = 0.2$ mm). In this way, the reconstructed droplet images are not affected by the coherent noise coming from the out-of-focus droplets that forced the use of the intensity correlation in the previous steps.

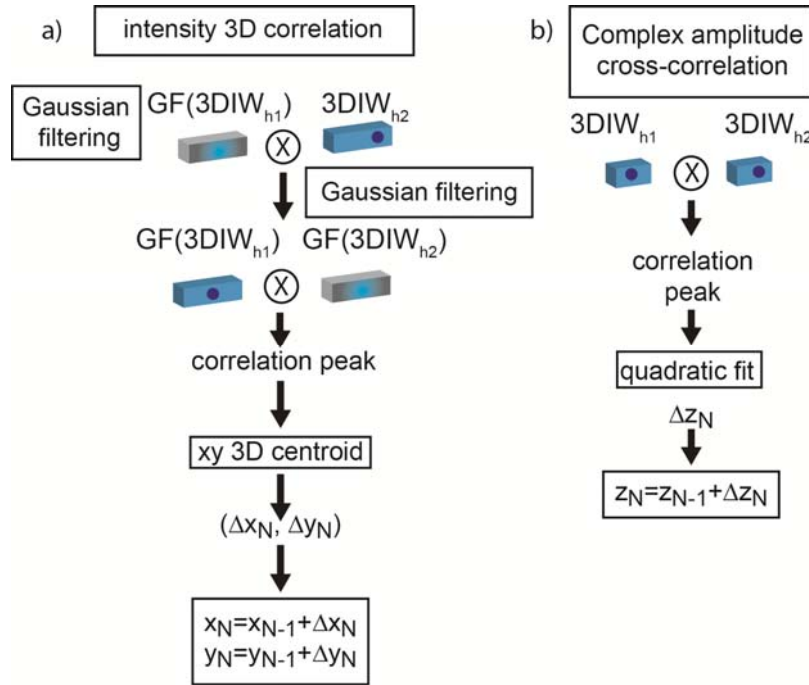


Figure 4: Droplet displacements calculation between the first and Nth holograms: a) first and second intensity cross-correlations to calculate the in-plane displacement; b) third complex amplitude cross-correlation to obtain the out-of-plane displacement.

Once the three coordinates are found in the second hologram, the droplet must be tracked along the hologram series to measure $(\Delta x_N, \Delta y_N, \Delta z_N)$. The Tracking From Beginning relies in the calculation of the 3D cross-correlation between one 3DIW in the first hologram, centered in the droplet, and a shifted 3DIW in the Nth hologram, but centered in the droplet position obtained from the (N-1)th hologram. In that way, a small 3DIW size can be kept, as if the cross-correlation

were performed between consecutive holograms, but avoiding the accumulative error that appears when consecutive recordings are used to follow a droplet.

5. Results

The analyzed flow was generated by spraying water into the air. Series of 1000 holograms were recorded with a time interval between two consecutive holograms $\Delta T = 1\text{ms}$. Figure 5 shows the evolution of the position of the droplets in three different times, near the spray injection. We can see how the droplets are expanding from the initial injection and are going down. Figure 6 shows the velocity vector map at $t_0+10\text{ms}$, where the color code corresponds to V_x . In this map we can see the influence of the cuvette walls.

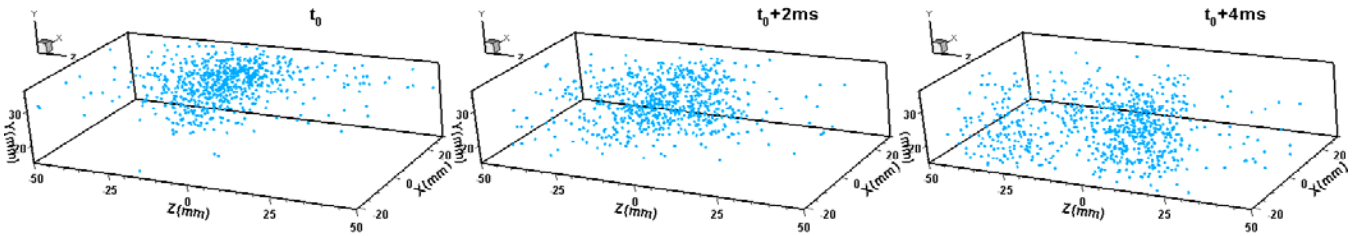


Figure 5: Spray droplet position at three different times

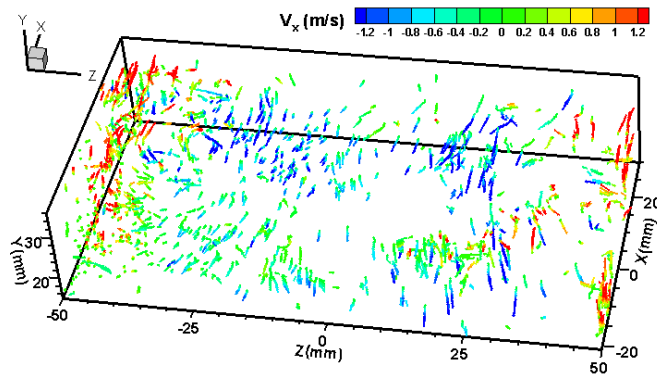


Figure 6: Spray droplet velocity.

6. Conclusions

In this work, we show the measurements done in an water aerosol, where we have recorded a field view twice the area allowed by the camera sensor, with a magnification $M=1$, with a very

large dimension along the optical axis. The great advantage of this technique is its very simple set-up, which requires a camera, a lens and a frequency filter to record the holograms.

A fully automatic algorithm allows us to determine the position of the droplets and its velocity. Its strength lies in the identification of the pixels that most contribute to the droplet image. This method allows us to analyze fluid volumes, independently of their axial dimension, by dividing them in appropriated sections. We have found that the algorithm performance is not affected by the frequency filter characteristic of the sideband configuration.

Droplets are tracked with a multi-pass, adaptive interrogation process called Adaptive Cross-Correlation Tracking From Beginning. In this method, small 3DIWs can be kept, as if the cross-correlation was performed between consecutive holograms, but avoiding the accumulative error that appears when consecutive recordings are used to follow a particle.

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